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Dynamical breaking and restoration of chiral and color symmetries for an accelerated observer and in the static Einstein universe

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Abstract
We investigate the formation of quark and diquark condensates in two different situations. First, we study the phenomenon of chiral and color symmetry breaking and their restoration for a uniformly accelerated observer due to the thermalization Hawking–Unruh effect. The gap equations for quark and diquark condensates with finite chemical potential are constructed. The critical value of acceleration is also obtained. Second, we consider the phase transitions in dense matter with quark and diquark condensates in the static Einstein universe at finite temperature and chemical potential. The nonperturbative expression for the thermodynamic potential is obtained. The phase portraits of the system are constructed.

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1. Introduction

Effective field theories with four-fermion interaction of the Nambu–Jona-Lasinio type (NJL) [1] are quite useful in describing the physics of light mesons and diquarks. It was proposed more than 20 years ago [2–4] that at high baryon densities a colored diquark condensate $\langle qq\rangle$ might appear. In analogy with ordinary superconductivity, this effect was called color superconductivity (CSC). The possibility for the existence of the CSC phase in regions of moderate densities was recently proved (see, e.g., papers [5–9]). Since quark Cooper pairing occurs in the color anti-triplet channel, a nonzero value of $\langle qq\rangle$ means that, apart from the electromagnetic $U(1)$ symmetry, the color $SU_c(3)$ symmetry should be spontaneously broken inside the CSC phase as well. In the framework of NJL models the CSC phase formation has generally been considered as a dynamical competition between diquark $\langle qq\rangle$ and usual quark–antiquark condensation $\langle\bar{q}q\rangle$. Special attention has also been paid to the catalyzing influence of color magnetic fields on the condensation of diquarks.

Recently, the dynamical chiral symmetry breaking and its restoration for a uniformly accelerated observer due to the thermalization effect of acceleration was studied in [10] at zero chemical potential. Further investigations of the possible influence of the Unruh temperature on the phase transitions in dense quark matter with a finite chemical potential, and especially on the restoration of the broken color symmetry in CSC are thus especially interesting.

Related problems have also been studied for chiral symmetry breaking in a curved spacetime [11, 12], which may be useful for the investigation of compact stars, where the gravitational field is strong and its effect cannot be neglected.

The paper is organized as follows. In the following section, we briefly review the extended NJL model in the curved spacetime and obtain the general expression for the effective potential in the mean-field approximation. Then we apply these formulae to two different situations. First, in section 3, we study the restoration of chiral and color symmetries for a uniformly accelerated observer by using a NJL-type model formulated in Rindler coordinates. We calculate quark and diquark condensates as functions of the Unruh temperature and finite chemical potential. We report the critical values of acceleration (the critical Hawking–Unruh temperatures) for the restoration of the broken chiral and color symmetries first obtained in [13]. Second, in section 4, we study the influence of gravitational field, temperature and chemical potential on the behavior of the quark and diquark condensates. As the simplest model of the curved spacetime we take the static Einstein universe. This model is widely used in the literature in studying the phenomenon of Bose–Einstein condensation (see, e.g. [14] and references therein). This allows us to derive a nonperturbative expression for the thermodynamical potential and to construct the phase portraits of our system (for more details see [15]).

2. The extended NJL model in the curved spacetime

In the D-dimensional curved spacetime with signature $(+,-,-,-,\dots)$, the line element is written as

$$ds^2 = \eta_{\hat{a}\hat{b}} e^{\hat{a}}_{\mu} e^{\hat{b}}_{\nu}\, dx^{\mu}\, dx^{\nu}.$$

The gamma-matrices γ_μ, metric $g_{\mu\nu}$ and the vielbein $e^{\mu}_{\hat{a}}$, as well as the definitions of the spinor covariant derivative ∇_ν and spin connection $\omega^{\hat{a}\hat{b}}_{\nu}$ are given by the following relations [16, 17]:

$$\{\gamma_\mu(x),\gamma_\nu(x)\} = 2g_{\mu\nu}(x), \qquad \{\gamma_{\hat{a}},\gamma_{\hat{b}}\} = 2\eta_{\hat{a}\hat{b}}, \qquad \eta_{\hat{a}\hat{b}} = \mathrm{diag}(1,-1,-1,-1,\dots),$$
$$g_{\mu\nu}g^{\nu\rho} = \delta^{\rho}_{\mu}, \qquad g^{\mu\nu}(x) = e^{\mu}_{\hat{a}}(x)e^{\nu\hat{a}}(x), \qquad \gamma_\mu(x) = e^{\hat{a}}_{\mu}(x)\gamma_{\hat{a}}. \tag{1}$$

$$\nabla_\mu = \partial_\mu + \Gamma_\mu, \qquad \Gamma_\mu = \tfrac{1}{2}\omega^{\hat{a}\hat{b}}_{\mu}\sigma_{\hat{a}\hat{b}}, \qquad \sigma_{\hat{a}\hat{b}} = \tfrac{1}{4}[\gamma_{\hat{a}},\gamma_{\hat{b}}],$$
$$\omega^{\hat{a}\hat{b}}_{\mu} = \tfrac{1}{2}\, e^{\hat{a}\lambda}\, e^{\hat{b}\rho}[C_{\lambda\rho\mu} - C_{\rho\lambda\mu} - C_{\mu\lambda\rho}], \qquad C_{\lambda\rho\mu} = e^{\hat{a}}_{\lambda}\partial_{[\rho}e_{\mu]\hat{a}}. \tag{2}$$

Here, the index $\hat{a}$ refers to the flat tangent space defined by the vielbein at spacetime point x, and the $\gamma^{\hat{a}}$ $(\hat{a} = 0, 1, 2, 3, \dots)$ are the usual Dirac gamma-matrices of Minkowski spacetime. Moreover γ_5 is defined, as usual (see, e.g., [17–19]), i.e. to be the same as in flat spacetime and thus independent of spacetime variables.

A schematic model that demonstrates the formation of the color superconducting phase is the extended NJL model with up- and down-quarks. This model may be considered as the low-energy limit of QCD. For the color group $SU(3)_c$ its Lagrangian takes the form

$$\mathcal{L} = \bar{q}[\mathrm{i}\gamma^{\mu}\nabla_\mu + \mu\gamma^0]q + \frac{G_1}{2N_c}[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2] + \frac{G_2}{N_c}[\mathrm{i}\bar{q}_c\varepsilon\epsilon^b\gamma^5 q][\mathrm{i}\bar{q}\varepsilon\epsilon^b\gamma^5 q_c]. \tag{3}$$

Here, μ is the quark chemical potential, $q_c = C\bar{q}^t$, $\bar{q}_c = q^t C$ are charge-conjugated bispinors (t stands for the transposition operation). The charge conjugation operation is defined, as usual (see, e.g., [17]), with the help of the operator $C = \mathrm{i}\gamma^{\hat{2}}\gamma^{\hat{0}}$, where the flat-space matrices $\gamma^{\hat{2}}$ and $\gamma^{\hat{0}}$ are used.

The quark field $q \equiv q_{i\alpha}$ is a doublet of flavors and triplet of colors with indices $i = 1, 2; \alpha = 1, 2, 3$. Moreover, $\vec{\tau} \equiv (\tau^1, \tau^2, \tau^3)$ denote Pauli matrices in the flavor space; $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$, $(\epsilon^b)^{\alpha\beta} \equiv \epsilon^{\alpha\beta b}$ are the totally antisymmetric tensors in the flavor and color spaces, respectively.

Next, by applying the usual bosonization procedure, we obtain the linearized version of the Lagrangian (3) with the collective boson fields σ, $\vec{\pi}$ and Δ,

$$\begin{aligned}\tilde{\mathcal{L}} = \bar{q}[\mathrm{i}\gamma^\mu\nabla_\mu + \mu\gamma^0]q - \bar{q}(\sigma + \mathrm{i}\gamma^5\vec{\tau}\vec{\pi})q - \frac{3}{2G_1}(\sigma^2 + \vec{\pi}^2) - \frac{3}{G_2}\Delta^{*b}\Delta^b \\ - \Delta^{*b}[\mathrm{i}q^t C\varepsilon\epsilon^b\gamma^5 q] - \Delta^b[\mathrm{i}\bar{q}\varepsilon\epsilon^b\gamma^5 C\bar{q}^t].\end{aligned} \tag{4}$$

The fields σ and $\vec{\pi}$ are color singlets, and Δ^b is a color anti-triplet and flavor singlet. Therefore, if $\langle\sigma\rangle \neq 0$, the chiral symmetry is broken dynamically, and if $\langle\Delta^b\rangle \neq 0$, the color symmetry is broken.

In the one-loop approximation, the partition function can be written as follows:

$$Z = \int [\mathrm{d}q][\mathrm{d}\bar{q}][\mathrm{d}\sigma][\mathrm{d}\vec{\pi}][\mathrm{d}\Delta^{*b}][\mathrm{d}\Delta^b]\exp\left\{\mathrm{i}\int \mathrm{d}^D x\sqrt{-g}\tilde{\mathcal{L}}\right\}, \tag{5}$$

where $g = \det|g_{\mu\nu}|$. In what follows, it is convenient to consider the effective action for boson fields, which is expressed through the integral over quark fields

$$\exp\{\mathrm{i}S_{\mathrm{eff}}(\sigma, \vec{\pi}, \Delta^b, \Delta^{*b})\} = N'\int [\mathrm{d}q][\mathrm{d}\bar{q}]\exp\left\{\mathrm{i}\int \mathrm{d}^D x\sqrt{-g}\tilde{\mathcal{L}}\right\}, \tag{6}$$

where

$$S_{\mathrm{eff}}(\sigma, \vec{\pi}, \Delta^b, \Delta^{*b}) = -\int \mathrm{d}^D x\sqrt{-g}\left[\frac{3(\sigma^2 + \vec{\pi}^2)}{2G_1} + \frac{3\Delta^b\Delta^{*b}}{G_2}\right] + S_q, \tag{7}$$

where S_q is the quark contribution.

In the mean-field approximation, the fields σ, $\vec{\pi}$, Δ^b and Δ^{*b} can be replaced by their groundstate averages: $\langle\sigma\rangle$, $\langle\vec{\pi}\rangle$, $\langle\Delta^b\rangle$ and $\langle\Delta^{*b}\rangle$, respectively. Let us choose the following ground state of our model:

$$\langle\Delta^1\rangle = \langle\Delta^2\rangle = \langle\vec{\pi}\rangle = 0,$$

and denote $\langle\sigma\rangle$, $\langle\Delta^3\rangle \neq 0$, by letters σ, Δ. Evidently, this choice breaks the color symmetry down to the residual group $SU_c(2)$.

The quark contribution has the following form:

$$\begin{aligned}S_q(\sigma, \Delta) = -\mathrm{i}\ln\mathrm{Det}[(\mathrm{i}\hat{\nabla} - \sigma + \mu\gamma^0)] - \frac{\mathrm{i}}{2}\ln\mathrm{Det}[4|\Delta|^2 \\ + (-\mathrm{i}\hat{\nabla} - \sigma + \mu\gamma^0)(\mathrm{i}\hat{\nabla} - \sigma + \mu\gamma^0)].\end{aligned} \tag{8}$$

Here, the first determinant is over spinor, flavor and coordinate spaces, and the second one is over the two-dimensional color space as well, and $\hat{\nabla} = \gamma^\mu\nabla_\mu$.

Let us find the effective potential of the model with the global minimum point that will determine the quantities σ and Δ. By definition $S_{\mathrm{eff}} = -V_{\mathrm{eff}}\int \mathrm{d}^D x\sqrt{-g}$, where

$$V_{\mathrm{eff}} = \frac{3\sigma^2}{2G_1} + \frac{3\Delta\Delta^*}{G_2} + \tilde{V}_{\mathrm{eff}}; \qquad \tilde{V}_{\mathrm{eff}} = -\frac{S_q}{v}, \qquad v = \int \mathrm{d}^D x\sqrt{-g}. \tag{9}$$

3. Accelerated observer

The physics for an accelerated observer can be described by transforming from Minkowski coordinates $(x^0, x^1, \vec{x}_\perp)$ to the Rindler coordinates $(\eta, \rho, \vec{x}_\perp)$ by means of the following coordinate transformation:

$$x^0 = \rho \sinh a\eta, \qquad x^1 = \rho \cosh a\eta, \qquad x^i = x^i \; (i = 2, 3),$$

where η is the time variable in Rindler coordinates and a is the acceleration.

The line element is given by the relation

$$\mathrm{d}s^2 = a^2\rho^2\,\mathrm{d}\eta^2 - \mathrm{d}\rho^2 - \mathrm{d}\vec{x}_\perp^2.$$

In what follows, we shall limit our consideration to the right Rindler wedge. The worldline of the observer is thus given in Rindler coordinates as

$$\eta(\tau) = \tau, \qquad \rho(\tau) = 1/a, \qquad \vec{x}_\perp(\tau) = \text{const}. \tag{10}$$

The quark contribution to the effective action can be written in the form

$$S_q = S_{q1} + S_{q2} = -\frac{\mathrm{i}}{2}\left[\operatorname{tr}\ln B_1^2 + 2\operatorname{tr}\ln B_2^2\right], \tag{11}$$

where we have summed over colors (leading to the factor 2 in the second term; the tr-operation does not include color indices any more) and

$$\begin{aligned} B_1^2 &= (-\mathrm{i}\gamma^\nu\nabla_\nu - \sigma - \mu\gamma^0)(\mathrm{i}\gamma^\mu\nabla_\mu - \sigma + \mu\gamma^0), \\ B_2^2 &= 4|\Delta|^2 + (-\mathrm{i}\gamma^\nu\nabla_\nu - \sigma + \mu\gamma^0)(\mathrm{i}\gamma^\mu\nabla_\mu - \sigma + \mu\gamma^0). \end{aligned} \tag{12}$$

Here the product of the relevant operators appearing in (12) can be represented in Rindler coordinates as

$$B^2 \equiv (-\mathrm{i}\gamma^\nu\nabla_\nu - \sigma)(\mathrm{i}\gamma^\mu\nabla_\mu - \sigma) = \frac{1}{\rho^2}\left[\frac{1}{a}\partial_\eta + \frac{1}{2}\gamma_{\hat{0}}\gamma_{\hat{1}}\right]^2 - \left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - (\vec{\gamma}\vec{\nabla})_\perp^2 - \sigma^2\right]. \tag{13}$$

In what follows, it is convenient to represent the operators B_1^2, B_2^2 in the basis of the solutions of the squared Dirac equation

$$B^2\Psi_{\vec{k}_\perp,j}(\eta, \vec{x}_\perp, \rho) = 0. \tag{14}$$

The solutions can be sought in the form

$$\Psi_{\vec{k}_\perp,j}(\eta, \vec{x}_\perp, \rho) = \mathrm{e}^{-\mathrm{i}aj\eta}\,\mathrm{e}^{\mathrm{i}\vec{k}_\perp\vec{x}_\perp}\psi_j(\rho), \tag{15}$$

and hence, with the consideration of (13), the function $\psi_j(\rho)$ is the solution of the second-order Bessel differential equation forming the basis of the Rindler modes (see, e.g., [20])

$$\left(\rho^2\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} + \rho\frac{\mathrm{d}}{\mathrm{d}\rho} - m^2\rho^2 + E_j^2\right)\psi_j(\rho) = 0, \tag{16}$$

where for the Rindler modes in the fermion sector, we have to take

$$m^2 = \vec{k}_\perp^2 + \sigma^2, \qquad E_j^2 = \left(j \pm \frac{\mathrm{i}}{2}\right)^2, \qquad 0 < j < +\infty. \tag{17}$$

Two independent solutions of equations (14), (16) can be obtained by using the projection operator $P_\pm = \frac{1}{2}(1 \pm \gamma_{\hat{0}}\gamma_{\hat{1}})$:

$$\psi_j^{(+)} = P_+\psi_j, \qquad \psi_j^{(-)} = P_-\psi_j.$$

Then the normalized solutions of (16) look as follows:

$$\psi_j^{(-)}(\rho) = \frac{\sqrt{(-2\mathrm{i}j-1)\cosh\pi j}}{\pi}K_{\mathrm{i}j+\frac{1}{2}}(m\rho),$$
$$\psi_j^{(+)}(\rho) = \frac{\sqrt{(+2\mathrm{i}j-1)\cosh\pi j}}{\pi}K_{\mathrm{i}j-\frac{1}{2}}(m\rho), \tag{18}$$

where K_ν is the Macdonald function (modified Bessel function).

These solutions $\langle\rho|\vec{k}_\perp, j, \pm\rangle = \psi^{(\pm)}_{\vec{k}_\perp,j}(\rho)$, for which we shall use the shorthand notation $\psi_j^{(\pm)}(\rho) \equiv \psi^{(\pm)}_{\vec{k}_\perp,j}(\rho)$, form a complete set of functions.

Taking the above formulae and the product of operators (13) into account, we obtain

$$B_2^2 = 4|\Delta|^2 + \frac{1}{\rho^2}\left[\frac{1}{a}\partial_\eta + \frac{1}{2}\gamma_{\hat{0}}\gamma_{\hat{1}}\right]^2 - \left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - (\vec{\gamma}\vec{\nabla})^2_\perp - \sigma^2\right]$$
$$+\left(\frac{\mu}{a}\right)^2\frac{1}{\rho^2} - 2\frac{\mu}{a\rho}[\gamma_{\hat{0}}\sigma - \mathrm{i}\gamma_{\hat{0}}(\vec{\gamma}\vec{\nabla})]. \tag{19}$$

For our further calculations, we also have to transform, in the same way, the product of operators in B_1^2:

$$B_1^2 = \frac{1}{\rho^2}\left[\frac{1}{a}\partial_\eta + \frac{1}{2}\gamma_{\hat{0}}\gamma_{\hat{1}} - \mathrm{i}\frac{\mu}{a}\right]^2 - \left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - (\vec{\gamma}\vec{\nabla})^2_\perp - \sigma^2\right]. \tag{20}$$

In order to find nonvanishing condensates σ and Δ, one should solve the gap equations:

$$\frac{\partial V_{\text{eff}}}{\partial\Delta^*} = 0, \qquad \frac{\partial V_{\text{eff}}}{\partial\sigma} = 0. \tag{21}$$

Moreover, by taking into account the fact that the position of the accelerated observer is defined in (10), we can put $\rho = 1/a$.

3.1. Chiral symmetry breaking

First, let us consider chiral symmetry breaking. In this section, unlike [10], a nonzero chemical potential μ will be taken into account.

Let us first assume that $\Delta = 0$. Then according to (11) and (13),

$$S_q = -\frac{3}{2}\mathrm{i}\,\mathrm{tr}\ln B^2$$

and the gap equation looks like

$$\sigma = -\frac{\mathrm{i}G_1\sigma}{\int \mathrm{d}^4x\sqrt{-g}}\,\mathrm{tr}\,\frac{1}{B^2}.$$

Going over to the momentum representation we obtain

$$\sigma = -\mathrm{i}G_1\sigma N_f\int\frac{\mathrm{d}k_0}{2\pi}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^2}\int_0^\infty\frac{\mathrm{d}\rho}{\rho}\langle\vec{k}_\perp, k_0, \rho|\frac{1}{B^2}|\vec{k}_\perp, k_0, \rho\rangle|_{\rho=a^{-1}},$$

where $N_f = 2$ is the number of flavors in our problem. With the use of the completeness relation of the Rindler basis $\langle\rho|\vec{k}_\perp, j, \pm\rangle = \psi^{(\pm)}_{\vec{k}_\perp,j}(\rho)$, let us go over to this basis in the variable ρ, so that

$$\rho^2\frac{\mathrm{d}^2}{\mathrm{d}\rho^2} + \rho\frac{\mathrm{d}}{\mathrm{d}\rho} - m^2\rho^2 \to -\left(j \pm \frac{\mathrm{i}}{2}\right)^2.$$

Next, we are going to an imaginary time coordinate, i.e., to the Euclidean spacetime in order to consider the thermal effect of acceleration [21, 22]. The Euclidean Rindler spacetime has a singularity at $\rho = 0$, therefore we have to choose the period of the imaginary time as $2\pi/a$ [16].

The gap equation for the critical curve, corresponding to $\sigma = 0$, finally looks as follows:

$$1 = \frac{G_1}{2\pi^2} N_f \int_0^\infty \mathrm{d}q\, q \left\{ \tanh\left(\pi \frac{q-\mu}{a}\right) + \tanh\left(\pi \frac{q+\mu}{a}\right) \right\}. \qquad (22)$$

The above equation precisely corresponds to the known expression for the critical curve, obtained for finite temperature and chemical potential (see, e.g. [23]), if the correspondence between the acceleration a and Unruh temperature T is taken into consideration,

$$\frac{\pi}{a} = \frac{1}{2T}.$$

Now, recall that the Unruh temperature is given by the relation

$$T = \frac{a}{2.5 \times 10^{22} (\mathrm{cm\ s^{-2}})} \mathrm{K}.$$

Let us take the value of the maximum critical temperature on the transition curve for the quark condensate formation $T_m = 0.169$ GeV, calculated in [23]. Then, we find for the critical acceleration the following estimate $a_c = 2\pi T = 2\pi \times 0.169\ \mathrm{GeV} = 3.2 \times 10^{35}\ \mathrm{cm\ s^{-2}}$. This value is an order of magnitude larger than the value found for the case of a vanishing chemical potential in [10].

3.2. Color symmetry breaking and formation of a diquark condensate

In order to study the minimum in the variable Δ of the diquark condensate, we may here put $\sigma = 0$. Now, we have

$$S_{q2} = -\mathrm{i}\,\mathrm{tr}\ln B_2^2 = -\mathrm{i}\,\mathrm{tr}\ln[4|\Delta|^2 + (-\mathrm{i}\gamma^\nu\nabla_\nu + \mu\gamma^0)(\mathrm{i}\gamma^\mu\nabla_\mu + \mu\gamma^0)]. \qquad (23)$$

The corresponding gap equation now takes the form

$$\frac{3\Delta}{G_2} = \frac{1}{\int \mathrm{d}^4x\sqrt{-g}} \frac{\partial S_{q2}}{\partial \Delta^*}.$$

Then the operators in the above equations can be expanded in the Rindler basis (18). Let us again estimate the value of the critical Unruh temperature and acceleration, at which the broken color symmetry is restored. For this purpose, we now put $\Delta = 0$, and after going over to the Euclidean spacetime and Matsubara frequencies the gap equation can again be written in the form

$$1 = \frac{2}{3}\frac{G_2}{\pi^2} N_f \left[\int_0^\Lambda \mathrm{d}q\, q^2 \frac{\tanh\frac{\pi(q+\mu)}{a}}{q+\mu} + \int_\mu^\Lambda \mathrm{d}q\, q^2 \frac{\tanh\frac{\pi(q-\mu)}{a}}{q-\mu} + \int_0^\mu \mathrm{d}q\, q^2 \frac{\tanh\frac{\pi(\mu-q)}{a}}{\mu-q} \right],$$

where the upper limit in the integral was replaced by the cutoff Λ for the physical regularization. If the correspondence $\frac{\pi}{a} = \frac{1}{2T}$ between the acceleration a and temperature T is taken into consideration, this result exactly corresponds to the well-known formula for the critical curve in the usual CSC theory at finite temperature [24, 23]. Let us again give a rough estimate of the order of the critical acceleration using the numerical results of [23]. By taking their value of the critical temperature on the transition curve for color superconductivity $T_c = 40$ MeV, and the chemical potential $\mu = 0.4$ GeV, we find for the critical acceleration the following estimate $a_c = 2\pi T_c = 2\pi \times 0.04\ \mathrm{GeV} = 7.5 \times 10^{34}\ \mathrm{cm\ s^{-2}}$, which differs from the critical acceleration for restoration of chiral symmetry by a factor 4.

4. Static Einstein universe

We will use the static D-dimensional Einstein universe as the simple example of the curved spacetime. The line element

$$ds^2 = dt^2 - a^2(d\theta^2 + \sin^2\theta\, d\Omega_{D-2}) \tag{24}$$

gives the global topology $\mathbb{R} \otimes \mathbb{S}^{D-1}$ of the universe, where a is the radius of the universe, related to the scalar curvature by the relation $R = (D-1)(D-2)a^{-2}$. The volume of the universe is determined by the formula

$$V(a) = \frac{2\pi^{D/2}a^{D-1}}{\Gamma\left(\frac{D}{2}\right)}. \tag{25}$$

In analogy with the case of flat Minkowski spacetime, we introduce the Hamiltonians $\hat{H}$ and $\hat{\mathcal{H}}$ of massless and massive particles respectively

$$\hat{H} = \vec{\alpha}\hat{\vec{p}}, \qquad \hat{\mathcal{H}} = \vec{\alpha}\hat{\vec{p}} + \sigma\gamma^0, \tag{26}$$

where $\alpha^k = \gamma^0\gamma^k$, and $(\hat{p})^k = -i\nabla_k, k = 1, \ldots, D-1$.

Then the quark contribution to the effective action can be written in the following form:

$$S_q = -\frac{i}{2}\left\{\ln \mathrm{Det}[\hat{\mathcal{H}}^2 - (\hat{p}_0 - \mu)^2] + \ln \mathrm{Det}\left[4|\Delta|^2 + (\hat{\mathcal{H}} - \mu)^2 - \hat{p}_0^2\right]\right\}, \tag{27}$$

where we have summed over colors (Det-operator does not include color space).

The eigenvalues of the operators $\hat{H}$ and $\hat{\mathcal{H}}$ may be found exactly for the case of the static D-dimensional Einstein universe. They are expressed through the corresponding eigenvalues of the Dirac operator on the sphere $\mathbb{S}^{D-1}$ [19, 25]

$$\begin{aligned} &\hat{H}\psi_l = \pm\omega_l\psi_l, \qquad \omega_l = \frac{1}{a}\left(l + \frac{D-1}{2}\right), \\ &\hat{\mathcal{H}}\psi_l = \pm E_l\psi_l, \qquad E_l = \sqrt{\omega_l^2 + \sigma^2}, \qquad l = 0, 1, 2 \ldots. \end{aligned} \tag{28}$$

The degeneracies of ω_l and E_l are equal to

$$d_l = \frac{2^{[(D+1)/2]}\Gamma(D + l - 1)}{l!\Gamma(D-1)}, \tag{29}$$

where $[x]$ is the integer part of x.

After going over to the Euclidian spacetime and summing over Matsubara frequencies we obtain the thermodynamic potential

$$\begin{aligned} \Omega(\sigma, \Delta) = 3\left(\frac{\sigma^2}{2G_1} + \frac{|\Delta|^2}{G_2}\right) &- \frac{N_f}{V}(N_c - 2)\sum_{l=0}^{\infty} d_l\{E_l + T\ln(1 + e^{-\beta(E_l-\mu)}) \\ &+ T\ln(1 + e^{-\beta(E_l+\mu)})\} - \frac{N_f}{V}\sum_{l=0}^{\infty} d_l\left\{\sqrt{(E_l-\mu)^2 + 4|\Delta|^2} + \sqrt{(E_l+\mu)^2 + 4|\Delta|^2}\right. \\ &\left. + 2T\ln\left(1 + e^{-\beta\sqrt{(E_l-\mu)^2+4|\Delta|^2}}\right) + 2T\ln\left(1 + e^{-\beta\sqrt{(E_l+\mu)^2+4|\Delta|^2}}\right)\right\}. \end{aligned} \tag{30}$$

Now, imposing the condition on the effective potential, $\Omega(0, 0) = 0$, we should subtract a corresponding constant from it. The thermodynamic potential, normalized in this way, is still divergent at large l, and hence, we should introduce a (soft) cutoff in the summation over l by means of the multiplier $e^{-\omega_l/\Lambda}$ [12], where Λ is the cutoff parameter.

In flat spacetime, the regularization cutoff constant Λ can be determined from the experimental results like pion mass or pion decay constant. Since we have no such experimental

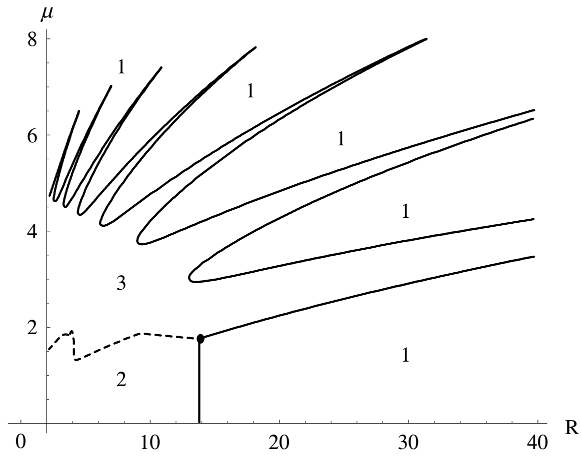

Figure 1. The phase portrait at $T = 0$ for $G_1 = 10$. Dashed (solid) lines denote first (second)-order phase transitions. The bold point denotes a tricritical point. The numbers 1, 2 and 3 designate the symmetric phase, the phase with chiral symmetry breaking and the superconducting phase, respectively.

data in the curved spacetime, we restrict ourselves only to qualitative study of gravitation effects. To this end, we scale the thermodynamic potential and all relevant quantities like condensates, curvature, chemical potential and temperature by the unknown cutoff Λ. The Cooper instability between quarks does not occur if the cutoff scale is less than the chemical potential. However, since we use the soft cutoff regularization we think that our results are still valid in the vicinity of the cutoff parameter as well.

Then the regularized thermodynamic potential is written as

$$
\begin{aligned}
\Omega^{\text{reg}}(\sigma, \Delta) = {} & 3\left(\frac{\sigma^2}{2G_1} + \frac{|\Delta|^2}{G_2}\right) - \frac{N_f}{V}(N_c - 2)\sum_{l=0}^{\infty} \mathrm{e}^{-\omega_l} d_l \\
& \times \{E_l + T\ln(1 + \mathrm{e}^{-\beta(E_l-\mu)}) + T\ln(1 + \mathrm{e}^{-\beta(E_l+\mu)})\} \\
& - \frac{N_f}{V}\sum_{l=0}^{\infty} \mathrm{e}^{-\omega_l} d_l \Big\{\sqrt{(E_l - \mu)^2 + 4|\Delta|^2} + \sqrt{(E_l + \mu)^2 + 4|\Delta|^2} \\
& + 2T\ln\Big(1 + \mathrm{e}^{-\beta\sqrt{(E_l-\mu)^2+4|\Delta|^2}}\Big) + 2T\ln\Big(1 + \mathrm{e}^{-\beta\sqrt{(E_l+\mu)^2+4|\Delta|^2}}\Big)\Big\}.
\end{aligned} \tag{31}
$$

In the following section, we shall perform the numerical calculation of the points of the global minimum of the finite regularized thermodynamic potential $\Omega^{\text{reg}}(\sigma, \Delta) - \Omega^{\text{reg}}(0, 0)$, and with the use of them, consider phase transitions in the Einstein universe.

4.1. Phase transitions

In what follows, we shall fix the constant G_2, similarly to what has been done in the flat case [6, 26], by using the relation $G_2 = \frac{3}{8}G_1$. For numerical estimates, let us choose the constant $G_1 = 10$ such that the chiral and/or color symmetries are completely broken. Moreover, let us now limit ourselves to the investigation of the case $D = 4$ only.

In figure 1, the R–μ-phase portrait of the system at zero temperature is depicted. For points in the symmetric phase 1, the global minimum of the thermodynamic potential is at $\sigma = 0, \Delta = 0$ (chiral and color symmetries are unbroken). In the region of phase 2, only

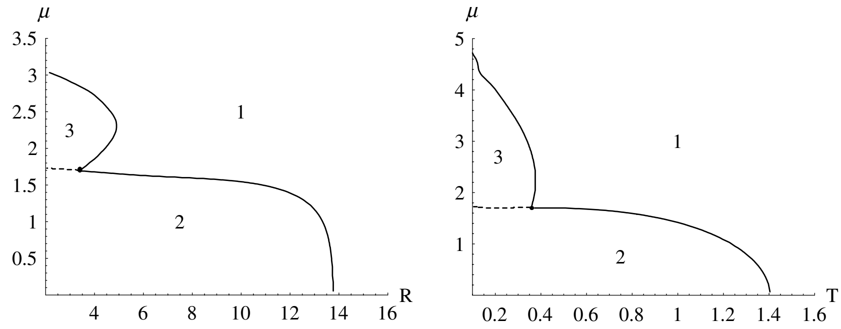

Figure 2. The phase portraits at $T = 0.35$ (left picture) and at $R = 3$ (right picture), $G_1 = 10$.

chiral symmetry is broken and $\sigma \neq 0, \Delta = 0$. The points in phase 3 correspond to the formation of the diquark condensate (color superconductivity) and the minimum takes place at $\sigma = 0, \Delta \neq 0$.

Moreover, the oscillation effect clearly visible in the phase curves of figure 1 should be mentioned. This may be explained by the discreteness of the fermion energy levels (28) in the compact space. This effect may be compared to the similar effect in the magnetic field H, where fermion levels are also discrete (the Landau levels).

In addition, we considered also phase transitions at finite temperatures. In figure 2, R–μ- and T–μ-phase portraits are depicted. It is clear from figure 2 that with growing temperature both the chiral and color symmetries are restored. The similarity of plots in R–μ and T–μ axes leads one to the conclusion that the parameters of curvature R and temperature T play similar roles in restoring the symmetries of the system.

5. Summary and conclusions

First, we have investigated the role of the thermalization effect by the acceleration of an observer for the restoration of chiral and color symmetries in quark matter at finite density in the framework of the NJL model. Obviously, the acceleration here plays the role of the temperature, as if the system is placed into a thermostat.

Second, we have considered the phase transitions in the static Einstein universe at finite temperature and chemical potential. The effects of gravitation were exactly taken into account. Moreover, an oscillation effect of the phase curves was found, which may be explained by the discreteness of the fermion energy levels in the compact space.

The dependence of chiral and color properties of the quark matter on the acceleration of the observer may be useful in the physics of black holes, where the Rindler metric can be considered as an approximation for the description of the surface gravitational fields. Moreover, the investigation of the influence of strong gravitational fields, such as in compact stars, on the diquark condensation and thus on the possible existence of color superconductivity in the core of the compact stars, is also of great importance.

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